PARAMETRIC IDENTIFICATION OF STATIC NONLINEARITIES IN A GENERAL INTERCONNECTED SYSTEM

Kenneth Hsu[†], Carlo Novara[°]
Mario Milanese[♯], Kameshwar Poolla[‡]

Abstract: We are concerned with the identification of static nonlinear maps in a structured interconnected system. Structural information is often neglected in nonlinear system identification methods. In this paper, we exploit *a priori* structural information and use parametric identification methods. We focus on the case where the linear part of the interconnection is known and only the static nonlinear components require identification. We propose an identification algorithm and investigate its convergence properties.

Keywords: system identification, nonlinear systems, structured systems, convergence, parametric

1. INTRODUCTION

This paper is concerned with identification problems in interconnected nonlinear systems. These problems are of considerable importance in the context of control, simulation, and design of complex systems.

There is available limited past work on the identification of such systems on a case-by-case basis. These include studies of Hammerstein and Wiener systems (Billings and Fakhouri, 1978),(Narendra and Gallman, 1966),(Pawlak, 1991). However, many of the simplest problems here remain open. For instance, the systematic inclusion of *a priori* structural information has been limited by the lack of a paradigm that is sufficiently general to incorporate such information.

[1] Supported in part by NSF under Grant ECS 03-02554 and by Ministero dell'Università e della Ricerca Scientifica e Tecnologica, Italy, under the Project "Robustness techniques for control of uncertain systems"

[†] Dept. of Mechanical. Engineering, University of California, Berkeley, CA 94720. e-mail: ken@me.berkeley.edu
[♯] Dip. di Automatica e Informatica, Politecnico di Torino, Italy. e-mail: mario.milanese@polito.it
[°] Dip. di Automatica e Informatica, Politecnico di Torino, Italy. e-mail: carlo.novara@polito.it
[‡] Corresponding author. Dept. of Mechanical Engineering, University of California, Berkeley, CA 94720. e-mail: poolla@jagger.me.berkeley.edu

We believe that the development of generalizations such as linear fractional transformations (LFT's) in the control systems literature (A. Packard, 1993),(Safonov, 1982), together with the advent of powerful, inexpensive computational resources offer the promise of significant advances in system identification for complex nonlinear systems.

Much of the available literature treats nonlinear system identification problems in extreme generality, for example using Volterra kernel expansions, neural networks, or radial basis function expansions (Billings and Fakhouri, 1978),(Boutayeb *et al.*, 1993),(Johansen, 1996),(Sjoberg *et al.*, 1995). However, it is our conviction that a completely general theory of nonlinear system identification will have little material impact on the many practical problems that are of interest. We believe that it is more beneficial to study specific classes of nonlinear system identification problems, devise appropriate *systematic* algorithms, and to study the behavior of these algorithms. This experience can then be collated together with experimental studies to develop a broader theory.

In general, both first principle laws and black-box model selection procedures result in only an approximate modeling of the involved phenomena. For example, the identification procedure may be subject to incorrect *a priori* information which can counteract the positive effects of correct known information. Evaluating the overall balance of these two effects on the identification error is a largely open problem for nonlinear systems. These considerations motivate the need for identification methods to incoroporate known structural information which is to a large extent considered to be correct. We offer a systematic framework based on linear fractional transformations to incorporate known structural information about the interconnected system. While there is occasional work that incorporates *a priori* structural information (Narendra and Gallman, 1966),(Stoica, 1981),(Vandersteen and Schoukens, 1997),(Milanese and Novara, 2003), this is not commonly the case.

In this paper, we present an algorithm for the identification of static nonlinear components in interconnected systems. In particular, we are concerned with problems in which the static nonlinear elements to be identified are *parametric*. We prove that the estimated nonlinearity converges asymptotically. The proof of convergence follows the presentation in Chapter 8 of (Ljung, 1999). We assume that the linear components of the interconnection are known. Note that the Hammerstein and Wiener systems are special cases of our formulation and under this assumption, the identification of these systems becomes trivial. However, the class of problems we consider involves more complex interconnections that can not be captured by the Hammerstein and Weiner formulations.

The remainder of this paper is organized as follows. In Section 2, we define the class of model structures under consideration. In Section 3, we motivate and present our identification algorithm. Section 4 contains our main convergence result. Section 5 provides conditions under which our estimate converges to the "true" nonlinearity. In Section 6, we offer illustrative examples. The proofs of our main results may be found online at `http://jagger.me.berkeley.edu/~ken` or by contacting the authors.

NOTATION

$\mathbb{R}^n$	standard Euclidean space
$u, y, w, \cdots$	vector-valued discrete-time signals (finite or infinite)
y_t	value of signal y at time t
$\mathcal{D}^L$	finite input-output data record $\mathcal{D}^L = \{u_t, y_t\}_0^{L-1}$
L	length of data record
e	noise signal
LTI	linear time-invariant
$\mathcal{L}$	linear time-invariant operator
$\mathcal{N}$	static nonlinear operator
$\{\phi^{[k]}(\cdot)\}_1^N$	set of nonlinear basis functions
$\theta \in \mathbb{R}^N$	vector of parameters to be identified
N	number of basis functions
z, w	input and output signals of $\mathcal{N}$
θ^{true}	"true" vector of parameters
$\hat{\theta}^L$	estimate of θ based on first L samples of data record

2. PROBLEM FORMULATION

We are concerned with the identification of static nonlinear maps in general structured interconnected systems. An example of the class of "structured" systems we consider is shown in Figure 1. Here, the static nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$ are to be identified. The (possibly unstable) LTI systems $\mathcal{L}_1$ and $\mathcal{L}_2$ are known. We have access to the (noisy) input-output data $\mathcal{D}^L = \{u_t, y_t\}_0^{L-1}$ and e is a noise signal.

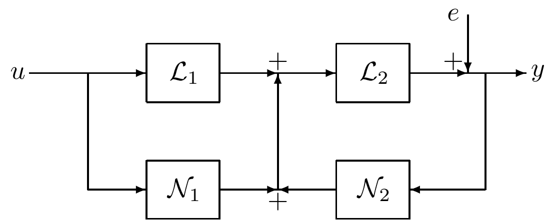

Fig. 1. Example of structured interconnected system

Any general interconnected nonlinear system may be represented through a linear fractional transformation (LFT) framework as shown in Figure 2. The LFT framework allows us to separate the LTI dynamics from the static nonlinearities in an interconnected system. The signals u, y are measured, and the signals z, w will denote the inputs and outputs of the static nonlinear block $\mathcal{N}$, respectively. The signal e is a zero-mean white noise process.

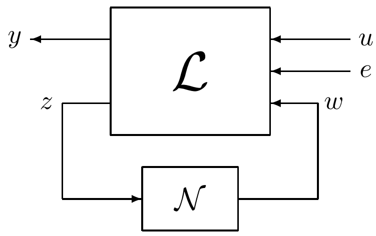

Fig. 2. LFT Model Structure

We gather all the nonlinearities of the interconnection into the multi-input multi-output block $\mathcal{N}$, which is to be identified. In general, the static nonlinear block $\mathcal{N}$ has block diagonal structure (Claassen, 2001).

$$\mathcal{N} = \begin{bmatrix} \mathcal{N}^{[1]} & & \\ & \ddots & \\ & & \mathcal{N}^{[m]} \end{bmatrix}$$

We partition the inputs z and outputs w of $\mathcal{N}$ conformably with its structure. More generally, the nonlinear block $\mathcal{N}$ may have *repeated* components. This situation arises when a particular nonlinearity appears more than once in the dynamical equations describing the interconnected system.

The frequently studied Hammerstein and Wiener systems are special cases of our formulation. However, under our additional assumption that the linear components of the interconnection are known, the identification of these classes of systems becomes trivial. Indeed, it is important to note that the class of problems we wish to identify involve *complex* interconnections. For example, consider the system depicted in Figure 3.

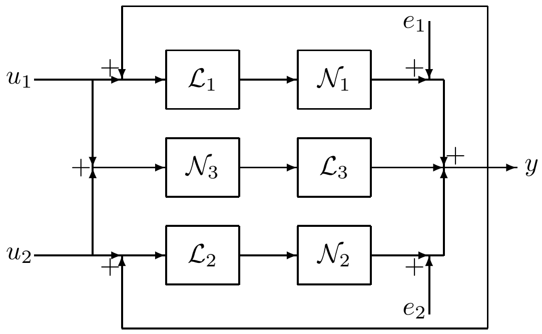

Fig. 3. Complexity in an interconnected system.

Here, the feedback interconnection along with the presence of numerous multivariable linear and nonlinear blocks suggests the complexity of the system. Figure 3 also illustrates a situation where the measured output is the sum of the outputs of several nonlinearites, as well as other signals. In order to develop a systematic approach for the identification of these systems, we use the LFT to collect all such systems under a common framework for analysis.

We will refer to the interconnected system of Figure 2 as the *LFT Model Structure*. We assume that the LTI block $\mathcal{L}$ and the dimensions of all signals is known. The components of the nonlinear block $\mathcal{N}$ are to be identified. For this, we have available measured (bounded) input-output data $\mathcal{D}^L = \{u_t, y_t\}_0^{L-1}$. We assume that the nonlinear block $\mathcal{N}$ can be parameterized by a finite set of nonlinear basis functions. By this, we mean that the input-output behavior of $\mathcal{N}$ can be described by

$$w = \mathcal{N}(z) = \sum_{k=1}^{N} \theta_k \phi^{[k]}(z), \quad \theta_k \in \mathbb{R} \tag{1}$$

In this paper we will address the problem of identifying the unknown parameters θ_k. Let us partition $\mathcal{L}$ comformably and realize it as

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_{yu} & \mathcal{L}_{ye} & \mathcal{L}_{yw} \\ \mathcal{L}_{zu} & \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} \sim \left[\begin{array}{c|ccc} A & B_u & B_e & B_w \\ \hline C_y & D_{yu} & D_{ye} & D_{yw} \\ C_z & D_{zu} & D_{ze} & D_{zw} \end{array}\right]$$

We summarize our principal assumptions below.

A.1 $\mathcal{L}$ has a stabilizable and detectable realization.
A.2 Measurability of z, i.e., there exists an LTI system Ψ_M such that

$$\begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} = \Psi_M \begin{bmatrix} \mathcal{L}_{ye} & \mathcal{L}_{yw} \end{bmatrix} \tag{2}$$

A.3 Co-measurability of z, i.e., there exists an LTI system Ψ_C such that

$$\begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} = \mathcal{L}_{zu} \Psi_C \tag{3}$$

A.4 $\mathcal{N}$ is parametric, i.e., $\mathcal{N}$ can be expressed as a finite sum of nonlinear basis functions

$$w = \mathcal{N}(z) = \sum_{k=1}^{N} \theta_k \phi^{[k]}(z), \quad \theta_k \in \mathbb{R}$$

A.5 There is no undermodelling. That is, there exists $\theta^{true} \in \mathbb{R}^n$ such that

$$w^{true} = \mathcal{N}^{true}(z) = \sum_{k=1}^{N} \theta^{true^{[k]}} \phi^{[k]}(z)$$

A.6 $\mathcal{L}_{yw} X \mathcal{L}_{ze}$ is strictly proper, where X is a matrix with the same block diagonal structure as $\mathcal{N}$, and $X^{[i]}$ has the same input-output dimensions as $\mathcal{N}^{[i]}$.
A.7 $D_{ye} D_{ye}^*$ is invertible.
A.8 We require

$$\lim_{L \to \infty} \frac{1}{L} \|\Phi(z)\|_I < \infty$$

where

$$\Phi(z) = \begin{bmatrix} \phi_1(z_0) & \cdots & \phi_N(z_0) \\ \vdots & \ddots & \vdots \\ \phi_1(z_{L-1}) & \cdots & \phi_N(z_{L-1}) \end{bmatrix}$$

and $\|\cdot\|_I$ is the induced norm:

$$\|\Phi(z)\|_I = \sup_{\theta \neq 0} \frac{\|\Phi(z)\theta\|}{\|\theta\|}$$

We now make several comments regarding these assumptions.

R.1 Note that we do not require $\mathcal{L}$ to be stable.
R.2 Assumption A.2 is critical to our needs. Observe that

$$\begin{aligned} z &= \mathcal{L}_{zu} u + \mathcal{L}_{ze} e + \mathcal{L}_{zw} w \\ &= \mathcal{L}_{zu} u + \Psi_M \mathcal{L}_{ye} e + \Psi_M \mathcal{L}_{zw} w \\ &= \mathcal{L}_{zu} u + \Psi_M (y - \mathcal{L}_{yu} u) \end{aligned}$$

This is equivalent to requiring that z be measured, i.e., z can be inferred from u, y and $\mathcal{L}$.
R.3 Assumption A.3 is the dual of assumption A.2. We do not require this assumption for our identification procedure. We require this only for our analysis on persistence of excitation (see Section 5.2).
R.4 Assumption A.4 is made to restrict $\mathcal{N}$ to the class of static nonlinearities that can be represented as a basis function expansion.
R.5 Assumption A.5 ensures that the behavior of the static nonlinear map $\mathcal{N}$ can be fully captured by our particular choice of basis functions.
R.6 A.6 is necessary so that the noise signal e is uncorrelated with $\mathcal{L}_{yw} w$. If $\mathcal{L}_{yw} w$ is not strictly proper, minimization of our cost function will result in bias in the estimates.

R.7 We require A.7 to ease computation of the Kalman Filter.

R.8 Assumption A.8 is made in order to guarantee convergence and uniqueness of the parameter estimate.

3. THE IDENTIFICATION ALGORITHM

In this section, we describe our proposed identification algorithm for general, structured interconnected nonlinear systems. In subsequent sections, we will analyze convergence properties of the proposed algorithm, address computational issues, and offer illustrative examples.

Let $B = [B_u \;\; B_e \;\; B_w]$ and $D = [D_{yu} \;\; D_{ye} \;\; D_{yw}]$. We now propose the following identification algorithm.

1. Perform stable coprime factorization
$\left[\mathcal{L}_{yu} \; \mathcal{L}_{ye} \; \mathcal{L}_{yw}\right] = G^{-1}\left[H_{yu} \; H_{ye} \; H_{yw}\right]$
where
$$G \sim \left[\begin{array}{c|c} \hat{A} & L \\ \hline C & I \end{array}\right], \quad H \sim \left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline C_y & D \end{array}\right]$$
are stabilized by L, and
$\hat{A} = A + LC_y, \quad \hat{B} = B + LD$

2. Realize the Kalman Filter $\mathcal{K}$ as
$$\mathcal{K} \sim \left[\begin{array}{c|c} F_t & K_t \\ \hline M_t & R_t \end{array}\right]$$
where
$$\begin{aligned} \hat{B}_e &= B_e + LD_e \\ K_t &= (\hat{A}P_tC_y^* + \hat{B}_eD_e^*)\Lambda_t^{-1} \\ F_t &= \hat{A} - K_tC_y \\ M_t &= \Lambda_t^{-\frac{1}{2}}C_y \\ R_t &= -\Lambda_t^{-\frac{1}{2}} \end{aligned}$$
and P_t, Λ_t, Z_t are obtained by solving the Riccati difference equation for $t = 0, \cdots, L-1$ with zero initial conditions $P_0 = 0$
$$\begin{aligned} P_{t+1} &= \hat{A}P_t\hat{A}^* + \hat{B}_e\hat{B}_e^* - Z_t\Lambda_tZ_t^* \\ \Lambda_t &= C_yP_tC_y^* + D_eD_e^* \\ Z_t &= (\hat{A}P_tC_y^* + \hat{B}_eD_e^*) \end{aligned}$$

3. Simulate Kalman Filter for $t = 0, \ldots, L-1$ with zero initial conditions to obtain
$$\begin{aligned} Y &= \mathcal{K}(Gy - H_{yu}u) \\ \mathcal{Q} &= \mathcal{K}H_{yw}\Phi(z) \end{aligned}$$

4. The parameter esitimate $\hat{\theta}^L$ is obtained by solving the convex least squares minimization problem
$\hat{\theta}^L = \arg\min_{\theta} \; J^L(\theta)$
where $J^L(\theta) = \frac{1}{L}\|Y - \mathcal{Q}\theta\|^2$

Fig. 4. Identification algorithm.

The rationale behind the algorithm can be explained as follows. We wish to choose the most likely signal e by solving the following optimization problem.
$$\max_{\theta} \; \mathbf{Prob}\{e|\theta\}$$
If assume e to be a white gaussian process, this is equivalent to choosing the minimum energy signal e. This can be accomplished with the following minimization problem.
$$\begin{gathered} \min_{\theta} \; \frac{1}{L}\|e\|^2 \quad \text{subject to} \\ \mathcal{L}_{ye}e = y - \mathcal{L}_{yu}u - \mathcal{L}_{yw}\sum_{k=1}^{N}\theta_k\phi^{[k]}(z) \end{gathered} \tag{4}$$
where the constraint restricts the signal e to be consistent with our input-output data. Note that if $\mathcal{L}_y = \left[\mathcal{L}_{yu} \; \mathcal{L}_{ye} \; \mathcal{L}_{yw}\right]$ is stable, we can choose
$$G = I \quad \text{and} \quad \left[H_{yu} \; H_{ye} \; H_{yw}\right] = \mathcal{L}_y$$
in step 1 of the algorithm. In the case that $\mathcal{L}_y$ is unstable, performing the coprime factorization allows us to rewrite (4) as
$$\begin{gathered} \min_{\theta} \; \|e\|^2 \quad \text{subject to} \\ H_{ye}e = Gy - H_{yu}u - H_{yw}\sum_{k=1}^{N}\theta_k\phi^{[k]}(z) \end{gathered} \tag{5}$$
in order to work with stable computations.

We recognize the minimization problem (5) as a Kalman Smoothing problem. Then, (5) is equivalent to the minimization problem in step 4 of the identification algorithm.

Note that by posing (5) as a Kalman Smoothing problem, we do not require the invertibility of $\mathcal{L}_{ye}$. Furthermore, this allows us to consider problems where the open loop system $\mathcal{L}$ may be unstable.

4. CONVERGENCE RESULTS

We now analyze the convergence properties of our candidate identification algorithm. We are interested in the asymptotic behavior of our estimate $\hat{\theta}^L$ as $L \to \infty$. Note that $\hat{\theta}^L$ is a random sequence because it depends on noisy measurements y and z.

Let us define the set of minimizers
$$\mathcal{M}^\infty = \left\{\hat{\theta} : \hat{\theta} = \arg\min_{\theta} \bar{J}(\theta)\right\}$$
where
$$\bar{J}(\theta) = \lim_{L\to\infty} \frac{1}{L}\|\mathcal{K}H_{yw}\Phi(z)(\theta^{true} - \theta)\|^2$$
Note that this limit exists due to Assumption A.8. Clearly, θ^{true} is a minimizer of $\bar{J}(\theta)$, i.e., $\theta^{true} \in \mathcal{M}^\infty$. Theorem 4.1 will show that the estimate $\hat{\theta}^L$ will converge to some $\theta \in \mathcal{M}^\infty$.

Theorem 4.1. Let Assumptions A.1,A.2,A.4-A.8 hold. Then,
$$\hat{\theta}^L \longrightarrow \mathcal{M}^\infty \quad \text{w.p. 1 as } L \to \infty$$
that is, $\lim_{L\to\infty} \|\hat{\theta}^L - \theta\| = 0$ with probability 1 for some $\theta \in \mathcal{M}^\infty$.

In the following section, we will provide conditions under which $\mathcal{M}^\infty = \{\theta^{true}\}$, i.e., $\mathcal{M}^\infty$ consists of the singleton θ^{true}.

5. IDENTIFIABILITY, PERSISTENCE OF EXCITATION, AND UNIQUENESS

5.1 Identifiability

In this section we discuss the notion of identifiability. Identifiability concepts are of fundamental importance in system identificaion (Ljung, 1999). Loosely speaking, the nonlinear block $\mathcal{N}$ of the LFT model structure is identifiable if it can be determined uniquely from input-output experiments.

Definition 5.1. Let $\mathbb{N}$ be the class of static nonlinearities that we consider. Let us represent the input-output behavior of the LFT model structure as $y = \Omega(\mathcal{L}, \mathcal{N})u$. Suppose $\mathcal{N}^o \in \mathbb{N}$. The LFT model structure $\Omega(\mathcal{L}, \mathcal{N})$ is *identifiable* at $\mathcal{N}^o$ if for any $\mathcal{N}^1 \in \mathbb{N}$ with $\mathcal{N}^1 \neq \mathcal{N}^o$,

$$\Omega(\mathcal{L}, \mathcal{N}^o) \neq \Omega(\mathcal{L}, \mathcal{N}^1)$$

The LFT model structure is *identifiable everywhere* if it is identifiable at all $\mathcal{N} \in \mathbb{N}$. □

Note that the above definitions are *global* notions of identifiability. For our case of parametric static nonlinearities, we have the following result on identifiability.

Lemma 5.2. The LFT model structure is identifiable everywhere if and only if

$$\mathcal{L}_{yw}\Phi(z)\theta = 0 \ \ \forall z \quad \Longrightarrow \quad \theta = 0$$

5.2 Persistence of Excitation

We now focus on the notion of persistence of excitation. The identifiability and persistence of excitation conditions are similar in that they are both necessary for the uniqueness of the solution to the parameter estimation problem. However, the two conditions apply to different aspects of the identification procedure. The identifiability condition is a condition on the structure of the model while the persistence of excitation condition is a condition on whether the input to the system is informative enough for our identification algorithm. The latter can be captured by the following definition.

Definition 5.3. The input z to the nonlinear block $\mathcal{N}$ is *persistently exciting* if and only if

$$\lim_{L\to\infty} \inf_L \frac{1}{L}\mathcal{Q}^*\mathcal{Q} \succ 0$$

for any set of linearly independent functions $\{\phi_k(\cdot)\}_1^N$.

Note that with the co-measurability assumption A.3, it is guaranteed that for any persistently exciting signal z, there exists an input u that could have generated z. This can be shown by the following. From Assumption A.2, we have that

$$\begin{aligned} z &= \mathcal{L}_{zu}u + \begin{bmatrix} \mathcal{L}_{ze} & \mathcal{L}_{zw} \end{bmatrix} \begin{bmatrix} e \\ w \end{bmatrix} \\ &= \mathcal{L}_{zu}u + \mathcal{L}_{zu}\Psi_C \begin{bmatrix} e \\ w \end{bmatrix} \end{aligned}$$

Then, $z \in Range(\mathcal{L}_{zu})$. That is, for any signals e, z, there always exists an input u that can generate z through $\mathcal{L}$ and $\mathcal{N}$.

5.3 Uniqueness

Theorem 5.4. Suppose Assumptions A.1,A.2,A.4-A.8 hold. If the LFT model structure is identifiable everywhere and z is persistently exciting, then

(1) There exists $L_0 > 0$ such that for any $L > L_0$, the minimization problem (5) yields the unique solution

$$\hat{\theta}^L = (\mathcal{Q}^*\mathcal{Q})^{-1}\mathcal{Q}^*Y$$

(2)

$$\mathcal{M}^\infty = \{\theta^{true}\}$$

Note that if $\mathcal{L}_{yw}$ is not strictly proper, then the cost function $J^L(\theta)$ will contain a correlation component which leads to bias in the estimate.

6. EXAMPLES

We now present two simulation examples demonstrating our identificaiton algorithm. Here, the input u was chosen to be a random sequence.

6.1 Example1

Consider the interconnected system in Figure 5. Here, the nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$ are to be identified. The LTI systems $\mathcal{L}_1$ and $\mathcal{L}_2$ are known, and the signals u, y are measured. The noise signal e is a white noise process acting on the output.

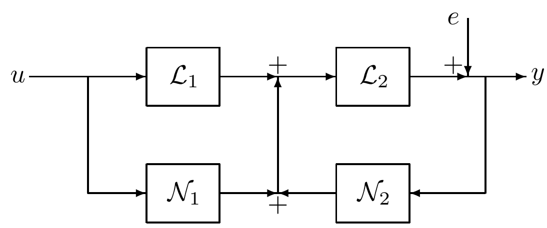

Fig. 5. Interconnected System for Example 1

The nonlinear block $\mathcal{N}$ of the LFT model structure is block diagonal, with $\mathcal{N}_1(u) = 2\arctan(u)$ and $\mathcal{N}_2(y) = -0.3y + 0.1\sin(y) + 1$. It is easy to verify that the input to the nonlinear block $z = [u \quad y]^T$ meets our measurability assumptions. Figure 6 illustrates the convergence behavior of our estimate. Here, the estimated parameters converge quickly to their true values as the size of the data sample increases.

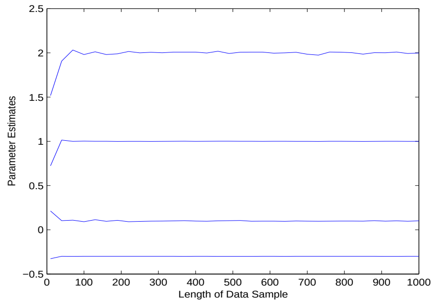

Fig. 6. Convergence of estimates for Example 1

6.2 Example 2

We now present an example where $\mathcal{L}$ is unstable. Consider the system depicted in Figure 7.

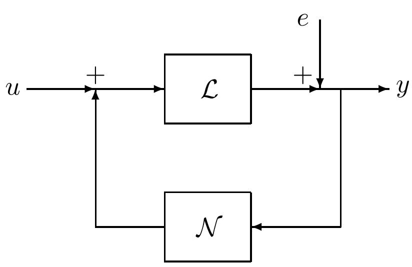

Fig. 7. Interconnected System for Example 2

Here, $\mathcal{L}$ is an unstable LTI system and we wish to identify $\mathcal{N}$. In this example, $\mathcal{N}^{true} = -0.25u + 5\arctan(u)$. Our identification algorithm allows us to consider systems where the linear block is unstable. Figure 8 illustrates the convergence behavior of our estimate for this example.

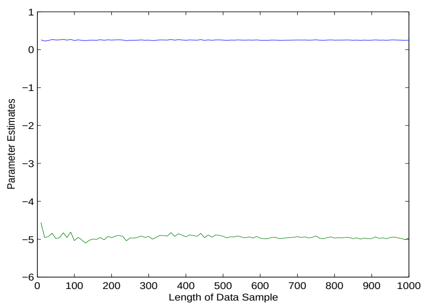

Fig. 8. Convergence of Estimates for Example 2

7. CONCLUSIONS

We have presented an algorithm for the identification of parametric static nonlinearites in interconnected systems. The interconnected systems can be represented through linear fractional transforms, which separates the linear and nonlinear components of the system. The LFT model structure also provides a common framework for the systematic inclusion of *a priori* structural knowledge in the identification algorithm. It was also shown that under certain conditions, the sequence of estimates converges asymptotically to the true value. These conditions consist of an identifiability condition on the LFT model structure and a persistence of excitation condition. Two numerical examples illustrated the effectiveness of the algorithm.

REFERENCES

A. Packard, J.C. Doyle (1993). The complex structured singular value. *Automatica* **29**, 71–110.

Billings, S.A. and S.Y. Fakhouri (1978). Identification of a class of nonlinear systems using correlation analysis. *Proc. of the IEEE* **125**, 691–697.

Boutayeb, M., M. Darouach, H. Rafaralahy and G. Krzakala (1993). A new technique for identification of miso hammerstein model. *Proc. of the ACC* **2**, 1991–1992.

Claassen, M. (2001). System identification for structured nonlinear systems. *Ph.D Dissertation, University of California at Berkeley.*

Johansen, T.A. (1996). Identification of nonlineawr systems using emperical data and prior knowledge-an optimization approach. *Automatica* **32**, 337–356.

Ljung, L. (1999). *System Identification Theory for the User, 2nd Edition.* Prentice Hall. Upper Saddle River, N.J.

Milanese, M. and C. Novara (2003). Structured experimental modeling of complex nonlinear systems. In: *Proc. of the 42nd IEEE Conference on Decision and Control.* Maui, Hawaii.

Narendra, K.S. and P.G. Gallman (1966). An iterative method for the identification of nonlinear systems using the hammerstein model. *IEEE TAC* **11**, 546–550.

Pawlak, M. (1991). On the series expansion approach to the identification of hammerstein systems. *IEEE TAC* **36**, 763–767.

Safonov, M.B. (1982). Stability margins of diagonally perturbed multivariable feedback systems. *IEEE Proc.* **129**, 251–256.

Sjoberg, J., Q. Zhang, L. Ljung, A. Benveniste, B. Delyon, P.Y. Glorennec, h. Hjalmarsson and A. Juditsdy (1995). Nonlinear black-box modeling in system identification: A unified overview. *Automatica* **31**, 1691–1724.

Stoica, P. (1981). On the convergence of an iterative algorithm used for hammerstein system identification. *IEEE TAC* **26**, 967–969.

Vandersteen, G. and J. Schoukens (1997). Measurement and identification of nonlinear systems consisting of linear dynamic blocks and one static nonlinearity. *IEEE Instrumentation and Measurement Technology Conference* **2**, 853–858.