Using the Polysys Class

This is a quick demonstration of the capabilities of the `@polysys` class.

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Contents

Creating a `polysys` object.

Since the `polysys` class is built on the `polynomial` class, we first create some `polynomial` objects to work with:

```
pvar x1 x2 u1 u2
```

The equations of the system are of the form

$$\dot{x} = f(x,u)$$

$$y = g(x,u)$$

Define the `polynomial` objects `f` and `g`

```
mu = -1;
f = [ x2; mu*(1-x1^2)*x2 - x1 ];
g = [x1;x2];
```

The `polynomial` objects `states` and `inputs` specify the ordering of the variables. For example, specifying `states(1)=x1` indicates that `f(1)` is the time derivative of `x1`.

```
states = [x1;x2];
inputs = [];
```

Finally, the `polynomial` objects are used to create a `polysys` object:

```
vdp = polysys(f,g,states,inputs)
```

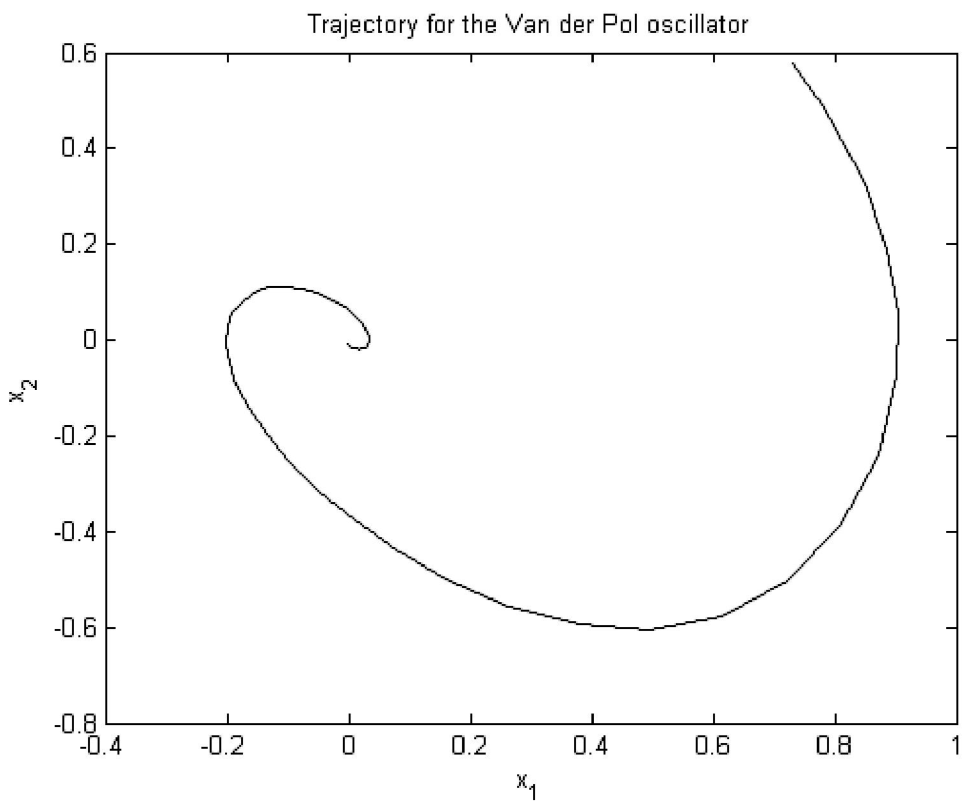
```
Continuous-time polynomial dynamic system.
```

```
States: x1,x2
State transition map is x'=f(x,u) where
  f1 = x2
  f2 = x1^2*x2 - x1 - x2
Output response map is y=g(x,u) where
  g1 = x1
  g2 = x2
```

Simulating the system.

The system is simulated over for a given time interval using the `sim` command. Note that the syntax is similar to `ode45`.

```
T = 10;
x0 = randn(2,1);
[t,x] = sim(vdp,[0,T],x0);

plot(x(:,1),x(:,2),'k-')
xlabel('x_1')
ylabel('x_2')
title('Trajectory for the Van der Pol oscillator')
```



Converting other objects to `polysys` objects.

The simplest object that can be "promoted" to a `polysys` is a `double`.

```
gainsys = polysys(rand(2,2))
```

```
Static polynomial map.
Inputs: u1,u2
Output response map is y=g(x,u) where
  g1 = 0.54722*u1 + 0.14929*u2
  g2 = 0.13862*u1 + 0.25751*u2
```

`LTI` objects can also be converted to `polysys` objects.

```
linearsys = rss(2,2,2);
linearpolysys = polysys(linearsys)
```

```
Continuous-time polynomial dynamic system.
States: x1,x2
Inputs: u1,u2
State transition map is x'=f(x,u) where
  f1 = -1.4751*u1 + 0.11844*u2 - 1.0515*x1 - 0.097639*x2
  f2 = -0.234*u1 + 0.31481*u2 - 0.097639*x1 - 1.9577*x2
Output response map is y=g(x,u) where
  g1 = 1.4435*x1 + 0.62323*x2
  g2 = -0.99209*u1 + 0.79905*x2
```

`Polynomial` objects can also be converted into a "static" `polysys` objects.

```
p = x1^2 - x1*x2;
staticsys = polysys(p)
```

```
Static polynomial map.
Inputs: u1,u2
Output response map is y=g(x,u) where
  g1 = u1^2 - u1*u2
```

Interconnections.

`Polysys` supports most of the same interconnections as the `LTI` class with the same syntax and the same semantics. Here are some examples:

```
append(linearpolysys,staticsys)
```

```
Continuous-time polynomial dynamic system.
States: x1,x2
Inputs: u1,u2,u3,u4
State transition map is x'=f(x,u) where
  f1 = -1.4751*u1 + 0.11844*u2 - 1.0515*x1 - 0.097639*x2
  f2 = -0.234*u1 + 0.31481*u2 - 0.097639*x1 - 1.9577*x2
Output response map is y=g(x,u) where
  g1 = 1.4435*x1 + 0.62323*x2
  g2 = -0.99209*u1 + 0.79905*x2
  g3 = u3^2 - u3*u4
```

```
series(linearpolysys,gainsys)
```

```
Continuous-time polynomial dynamic system.
States: x1,x2
Inputs: u1,u2
State transition map is x'=f(x,u) where
  f1 = -1.4751*u1 + 0.11844*u2 - 1.0515*x1 - 0.097639*x2
  f2 = -0.234*u1 + 0.31481*u2 - 0.097639*x1 - 1.9577*x2
Output response map is y=g(x,u) where
  g1 = -0.14811*u1 + 0.78991*x1 + 0.46034*x2
  g2 = -0.25547*u1 + 0.20011*x1 + 0.29216*x2
```

The methods `append`, `feedback`, `parallel`, and `series` are used to interconnect `polysys` objects.

Discrete-time systems.

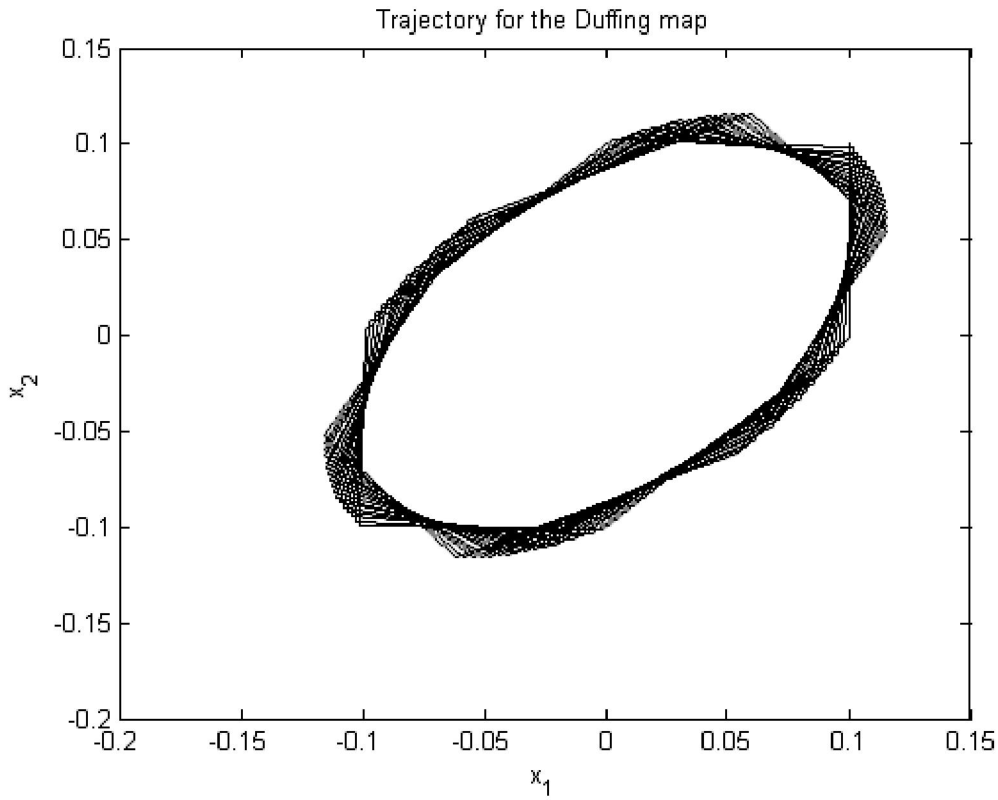
It is also possible to create discrete-time `polysys` objects, as follows:

```
a = 1;
b = 1;
fduff = [ x2; -b*x1 + a*x2 - x2^3 ];
gduff = [ x1; x2 ];

xduff = [ x1; x2];
uduff = [];
Tsample = 1;

duff = polysys(fduff,gduff,xduff,uduff,Tsample)
```

```
Discrete-time polynomial dynamic system.
Sampling time: 1
States: x1,x2
State transition map is x(k+1)=f(x(k),u(k)) where
  f1 = x2
  f2 = -x2^3 - x1 + x2
Output response map is y(k)=g(x(k),u(k)) where
  g1 = x1
  g2 = x2
```

Discrete-time systems are simulated using the command `dsim`. Note that simulation time points are specified as `(0:T)`, rather than `[0,T]`.

```
T = 100;
x0 = [.1;.1];
[t,x] = dsim(duff,(0:T),x0);

plot(x(:,1),x(:,2),'k-')
xlabel('x_1')
ylabel('x_2')
title('Trajectory for the Duffing map')
```



Other Utilities

`Polysys` object can be linearized at a given point. This syntax returns an `SS` object:

```
xe = [1;2];
vdplin = linearize(vdp,xe);
class(vdplin)
```

```
ans =
ss
```

This syntax returns the state-space data of the linearization:

```
[A,B,C,D] = linearize(vdp);
```

Check if a `polysys` object is linear.

```
islinear(linearpolysys)
```

```
ans =
     1
```

```
islinear(vdp)
```

```
ans =
     0
```

Subsystems are referenced using the same syntax as `LTI` objects:

```
linearpolysys(1,1)
```

```
Continuous-time polynomial dynamic system.
States: x1,x2
Inputs: u1
State transition map is x'=f(x,u) where
  f1 = -1.4751*u1 - 1.0515*x1 - 0.097639*x2
  f2 = -0.234*u1 - 0.097639*x1 - 1.9577*x2
Output response map is y=g(x,u) where
  g1 = 1.4435*x1 + 0.62323*x2
```

We can also get function handles to the system's state transition and output response maps. This is mostly used to build simulation routines that require handles to functions with a certain syntax (i.e., `ode45`).

```
[F,G] = function_handle(vdp);

xeval = randn(2,1);
ueval = []; % VDP is autonomous
teval = []; % The time input is just for compatibility with ode solvers
xdot = F(teval,xeval,ueval)
```

```
xdot =
   -0.7420
    0.9962
```

We can multiply `polysys` objects by scalars or matrices.

```
M = diag([2,3]);
M*vdp
```

```
Continuous-time polynomial dynamic system.
States: x1,x2
State transition map is x'=f(x,u) where
  f1 = x2
  f2 = x1^2*x2 - x1 - x2
Output response map is y=g(x,u) where
  g1 = 2*x1
  g2 = 3*x2
```

```
12*vdp
```

```
Continuous-time polynomial dynamic system.
States: x1,x2
```

```
State transition map is x'=f(x,u) where
  f1 = x2
  f2 = x1^2*x2 - x1 - x2
Output response map is y=g(x,u) where
  g1 = 12*x1
  g2 = 12*x2
```

```
linearpolysys*M
```

```
Continuous-time polynomial dynamic system.
States: x1,x2
Inputs: u1,u2
State transition map is x'=f(x,u) where
  f1 = -2.9503*u1 + 0.35533*u2 - 1.0515*x1 - 0.097639*x2
  f2 = -0.46801*u1 + 0.94443*u2 - 0.097639*x1 - 1.9577*x2
Output response map is y=g(x,u) where
  g1 = 1.4435*x1 + 0.62323*x2
  g2 = -1.9842*u1 + 0.79905*x2
```